

Unit 6: Rational Functions

Mary Margaret Sanford

Algebra II

Grades: 9-11

Unit 6: Rational Functions

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Teaching Context

School's Standardized testing Data History:

- ACT: B School (82.5)
 - Average of 20.5
 - 52% score at least 21 on ACT or gold on Work Keys
- Assessment: B School (85.1)
- Leap: 57% at mastery and above
- Graduation Rate: 85%
 - Grads earning credentials: 50%
 - College Enrollment Rate: 67%

Location: Suburban

School size: 1422 students

SES of the school population: everyone on free lunch

- 77% submit a FASFA

Classroom Student Information

- IEP/Act 833: 1
- 504: 14
- ESL: 1
- Gender
 - Female: 59
 - Male: 74
- Race
 - African American: 30
 - Caucasian: 96
 - Hispanic/Latino: 3
 - Asian: 1

School Demographics

Male: 743

Female: 679

Caucasian: 72.6%

African American: 20.2%

Hispanic: 5.2%

Asian: 1.5%

Native American: .4%

Big Ideas/Essential Questions

Big Ideas: Rational Expressions can be used to express properties of the physical world

Essential Questions:

- How would you define an asymptote?
- Why do you need to identify a hole vs an asymptote?
- Why do you need to find the GCD of the denominator in order to add/subtract a rational function?
- What does the solution of a rational equation tell you?
- What real-world scenario would require dividing two polynomials?
- How can we use rational expressions to explain the process of gravity?
- How are rational polynomials and modular arithmetic connected?

Content Standards

LSMM.A2:A-APR.D.6 - Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)+r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system

LSMM.A2:A-REI.A.2 - Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

LSSM.A2:F-IF.B.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

LSSM.A2:F-IF.C.7 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

CCSS.A2:APR.D.7 - Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Unit Overview

This unit is focused on rational expressions and equations. It is meant to be taught in an Algebra II classroom. Prior content required for this unit is a firm understanding of fractions and how they relate to each other, factoring, and basic rules of algebra. It is also suggested that students understand definitions of: horizontal and vertical asymptotes, domain and range, and degree of a function. Throughout this unit, students will be asked to identify vertical asymptotes, horizontal asymptotes, find domain and range, graph, add, subtract, multiply, divide, and solve rational expressions/equations. Rational expressions are important because of their use in Calculus, and later maths. Students will see rational expressions in many computational forms and they are also related to the real world. Rational expressions are greatly useful in

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equations involving gravity, work, and various other properties of the physical world. Students will see rational expressions in many other disciplines outside of math as well, with physics being the largest user. Below I have defined some of the words and phrases I will use throughout the Unit Plan, as a reflection of how these terms are used in my current teaching classroom.

Definitions and Explanations

Throughout this Unit Plan, I will be using a variety of terms which may not seem fitting to my plan. The classroom I teach in has a variety of terms and phrases to categorize what type of assignments the students will work on during a particular class period. Below, I will explain the various terms and phrases that I will be utilizing both in my unit plan and in my lesson.

The first term that I plan to use often is **recall**. A recall is a formative assessment that takes place, either as a whole group or small group, where the students review material that they learned previously in the unit, or from previous classes. **PKN** is the next acronym that I plan to use in my lessons. PKN stands for “Previous Knowledge Needed.” This could be placed in the SOW video or as separate instruction. A **SOW video** is an instructional video created by the teacher that the students engage in individually. The teachers use the word SOW because the teachers are planting a seed of knowledge in the students brain. The students engage in these videos individually so they can control when to pause, rewind, or slow down the pace. The videos are posted to Google Classroom so the students can re-watch the video at a later date, if necessary. Students are encouraged to engage in the video before class as “pre-work.”

A **ground sample** is another formative assessment utilized in the classroom. This is a one-on-one teacher and student understanding check that occurs after the students engage in the video. It is titled “ground sample” because it is a check to make sure the seed from the SOW video has been planted in good soil. Another formative assessment that will be utilized in class is **solution stations**. These are completed by the students so that they can practice problems on their own, or collaboratively. The teacher makes the solution stations on a PowerPoint slide and displays the problems on the projector; the solutions are pasted on the walls of the room for the students to self-assess. Some other formative assessments used in class are the platforms Socrative and Desmos.

An **irrigation** is whole group instruction based on the results of the formative assessments. This tends to be in the form of answering questions the students are struggling with. The term “irrigation” was coined because the students’ knowledge is being “watered from an outside source to continue to grow” (a quote pulled from my mentor teachers). A **campfire** is the term used when the teacher utilizes small group instruction based on formative assessments. For example, if a small group of students is struggling with a particular problem, they will come to the board and have it explained to them by the teacher. A **reap check** is a formal formative assessment given to the students at the end of class, which assesses the students on standards covered during class. The students are allowed 2 minutes to individually write whatever they like on a sticky note before the assessment begins. The students are allowed to take the reap check two times and they receive the higher grade. The title “reap” was chosen because it is a check to see if the students have reaped what was sown. Students are allowed to retake the assessment to receive a higher grade at a later date.

At the end of a unit there is a **review reap** which is a compilation of all reap check questions for a unit. It is available to students for unlimited attempts to receive extra practice and prepare for the

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SMHYG. The **SMWYK** (“Show Me What You Know”) is another way to prepare the students for the SMHYG. It is a formative assessment on all Unit Standards. The final way to prepare for the SMHYG are **grow stations**. These are extra practice questions to assist the students in preparation for the unit test. Students focus only on what they need to work on based on their results of the SMWYK. The final component of a unit is **SMHYG** (“Show Me How You Grow”), which is a formal, summative assessment on the unit. During the SMHYG, the students are tested on all unit standards and ACT standards practiced throughout the unit.

Content Outline

Introduction: What is a rational function?

- Quotient of two polynomials
- Of the form: $\frac{P(x)}{Q(x)} \longrightarrow \frac{\text{numerator}}{\text{denominator}}$
- Equations representing direct, inverse, and joint variation are examples of rational formulas that can model real-life situations.
- Examples of rational functions
 - Density formula
 - Rational equations can be used to solve a variety of problems that involve rates, times and work. Using rational expressions and equations can help you answer questions about how to combine workers or machines to complete a job on schedule
 - You can calculate how long it will take one person to do a job alone when you know how long it takes people working together to complete the job.

6.1: Vertical Asymptotes and Domain of Rational Functions

- To find vertical asymptotes, the denominator is factored out. Each factor is then set equal to zero to see where the x-intercepts of the graphs are located.
 - From here the student will be able to determine that the domain is all real numbers except for the zeros
- If there is a factor that is in the numerator and denominator, then the graph has a hole in it at that factor
- The domain of a rational function is all real numbers that don't break the function
 - Vertical asymptotes and holes break the function
- X-intercept: the x-intercepts of a rational function $\frac{P(x)}{Q(x)}$ (if there are any) are the numbers $\alpha \in \mathfrak{R}$ where $\frac{P(\alpha)}{Q(\alpha)} = 0$
- In Calculus, a vertical asymptote occurs when all is true:
 - $f(c)$ is undefined
 - $\lim_{x \rightarrow c^-} f(x) = \infty$ or $-\infty$
 - $\lim_{x \rightarrow c^+} f(x) = \infty$ or $-\infty$

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6.2: Horizontal Asymptotes (Honors: Slant) of Rational Functions

- To find horizontal asymptotes, the students need to look at the numerator and denominator. If the numerator and denominator have the same degree, then divide the leading coefficients to find the location of the horizontal asymptote
 - If the numerator has a lower degree than the denominator, then the x-axis is the horizontal asymptote
 - If the numerator has a higher degree than the denominator, then there is a slant asymptote (HONORS)
- (HONORS) Slant asymptotes occur when the numerator has a higher degree than the denominator
 - This means that the value of the numerator is stronger than the denominator, resulting in a graph that pulls away from the x-axis, causing a slant
 - To determine the slant, the students need to use long division
 - The resulting equation of the line (without the remainder) will give the students the line at which the slant asymptote occurs
- In Calculus, a horizontal asymptote occurs at the line $y=L$ if one of the following occurs:
 - $\lim_{x \rightarrow \infty} f(x) = L$
 - $\lim_{x \rightarrow -\infty} f(x) = L$
 - In order to find horizontal asymptotes in Calculus, one must evaluate limits as x approaches infinity
 - First, divide each of the terms in the numerator and denominator by the variable with the highest power
 - Next, evaluate the individual limits using the following rule: if r is a rational number greater than zero such that x^r is defined for all x , then $\lim_{x \rightarrow \pm\infty} \frac{1}{x^r} = 0$
- (HONORS) Slant (Oblique) Asymptotes in Calculus
 - If $\lim_{x \rightarrow \infty} [f(x) - (mx + b)] = 0$ then the line $y = mx + b$ is called the oblique or slant asymptote because the vertical distances between the curve $y = f(x)$ and the line $y = mx + b$ approaches 0

6.3: Graphing Rational Functions

- The first step to graphing is to find the intercepts
 - The x intercepts are determined by setting the numerator equal to zero and factor out the terms. Set the terms equal to zero to find the x-intercepts
 - To find the y-intercepts, set $x = 0$ and simplify the fraction
- Next, one needs to find the Vertical and Horizontal Asymptotes
 - To find the vertical asymptotes, set the denominator equal to zero and find the x-values
 - To find the horizontal asymptotes, the student will need to determine the degree of the function in the numerator and denominator. Whichever has a higher degree will tell the student where the horizontal asymptotes are located
- The graphs cannot cross horizontal and vertical asymptotes

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- To see what is around the asymptotes students would need to look at end behavior of the function by looking at the degree and leading coefficient
- The next step is to find the domain and range of the function
 - The domain is determined using the vertical asymptotes
 - The range is determined by finding the domain of the inverse function
 - To find the inverse of a function, one needs to simply flip x and y and solve for y
 - Once finding the inverse of the original rational function, the domain of the inverse will be the range of the original

6.4: Multiply and Divide Rational Functions

- When multiplying rational functions, the numerators of both the monomials need to be multiplied to each other, and the denominators need to be multiplied to each other.
- In the case of multiplying binomials, the process of FOIL is necessary
- Students should note that, when multiplying binomials/polynomials in factored form, one can cancel like terms if they are in the numerator and denominator
- When dividing rational functions, it is necessary to flip the numerator and denominator and multiply
- The difference between multiplying and dividing rational numbers vs expressions is, again, that instead of multiplying/dividing numbers, the students are asked to multiply/divide polynomials
 - When multiplying across the numerator and denominator, students will be asked to use the FOIL method in order to find the solution
 - It is a lot more complicated than simply multiplying two numbers together
 - Once students multiply numerators and denominators, if they were to put the terms in their factored forms, there is usually a term or two that can be canceled out (similar to simplifying a rational number)

6.5: Add and Subtract Rational Functions

- The first step when adding and subtracting rational functions is to simplify by canceling terms that are in both the numerator and denominator
 - This can be done through observation or through factoring
 - Students need to realize that you cannot simplify a term unless it is being multiplied to the rest of the term → given $\frac{x-7}{x}$ you cannot cancel the x
- The next step is to find common denominators using the GCD method
- Adding and subtracting rational numbers is different from adding and subtracting rational expressions because of the polynomial component
 - Finding the GCD of a two rational numbers is simple because they just have to have the same number in the denominator
 - When it is a rational expression, instead of having the same number in the denominator, the expression has to have the same polynomial, or group of factors in the denominator.

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- This makes things a little more complicated for students, as they will have to multiply each part of the expression likely by a factor or polynomial instead of a number in order to obtain a GCD.
- Real world example of adding and subtracting rational functions
 - Many work problems can be represented by the equation $\frac{t}{a} + \frac{t}{b} = 1$ where t is the time to do the job together, a is the time it takes person A to do the job, and b is the time it takes person B to do the job. The “1” refers to the total work done—in this case, the work was to paint “1” house.

6.6: Solving Rational Equations

- When solving rational functions, the student needs to cancel the denominators of each side by means of multiplication
- The method of solving the problem is given below
 - Given $\frac{a}{b} = \frac{c}{d}$ (where a, b, c, and d are rational expressions)
 - Multiply “b” to both sides of the equation to obtain $a = \frac{c}{d}b$
 - Then multiply “d” to both sides of the equation to obtain $ad = cb$
 - From here the students should be able to multiply the two expressions together and simplify where possible in order to solve for x
- If students put polynomials in factored form, they could notice that there are terms which can be simplified (if possible)

Unit Calendar

	Monday	Tuesday	Wednesday	Thursday	Friday
	March 16 Day: A Show Me How You Grow Activity (Pretest) 6.1: Discovery and SOW Video	-	March 18 Day: B Show Me How You Grow Activity (Pretest) 6.1: Discovery and SOW Video	March 19 Day: A 6.1: Vertical Asymptotes and Domain 6.2: Horizontal Asymptotes SOW Video	March 20 Day: B 6.1: Vertical Asymptotes and Domain 6.2: Horizontal Asymptotes SOW Video
Assessments	Pretest		Pretest	Reap 6.1	Reap 6.1
	March 23 Day: C 6.2: Horizontal Asymptotes	March 24 Day: A 6.3 Graphing 6.4: Sow Video for Multiplying and	March 25 Day: B 6.3 Graphing 6.4: Sow Video for Multiplying and	March 26 Day: A 6.4: Multiply and Dividing	March 27 Day: B 6.4: Multiply and Dividing

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		Dividing Rational Functions	Dividing Rational Functions		
Assessments	Reap 6.2	Reap 6.3	Reap 6.3	Reap 6.4	Reap 6.4
	March 30 Day: C 6.5: Adding and Subtracting Rational Functions	March 31 Day: A 6.5: Adding and Subtracting Rational Functions	April 1 Day: B 6.5: Adding and Subtracting Rational Functions	April 2 Day: A 6.6: Solving	April 3 Day: B 6.6: Solving
Assessments		Reap 6.5	Reap 6.5	Reap 6.6	Reap 6.6
	April 6 Day: A Show Me What You Know	April 7 Day: B Show Me What You Know	April 8 Day: A Show Me How You Grow	April 9 Day: B Show Me How You Grow	-
Assessments			Unit Test/Post Test	Unit Test/Post Test	

Possible Activities

Day 1/2: Vertical Asymptotes

- The students will spend the first 45 minutes of class taking a pre-assessment on the unit
- Students will watch a SOW Video on vertical asymptotes
- They will then complete a Socrative Assignment where they identify vertical asymptotes of functions
- There will be a Reap check at the end of the day where they will be asked to identify vertical asymptotes of a function

Day 3: Horizontal Asymptotes

- The students will engage in another SOW video on horizontal asymptotes
- They will then work on a desmos activity where the students will compare the differences of horizontal and vertical asymptotes
- They will then engage in a solution station/socrative assignment where they will be asked to identify whether the function as a horizontal or vertical asymptote, or both

Day 4/5: [Marbleslides: Rationals \(Desmos\)](#)

- This activity will be useful as discovery on the day of graphing.
- At this point the students should be able to identify what creates a vertical and horizontal asymptote
- This activity could allow the students to explore how to adjust a horizontal and vertical asymptote, thus giving them an opportunity to gain further knowledge on the subjects
- This could also allow the students to explore adding and subtracting rational functions

Day 6/7: Adding and Subtracting

- The students will watch the SOW video for this lesson during the previous class period

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- For this class period the students will work on practice problems with adding and subtracting rational functions

Day 8/9/10: Multiplying and Dividing

- The students will work on a socratic of multiplying and dividing rational functions
- Students will complete a maze activity on multiplying and dividing rational functions
- The solution to their problem will bring them to a new problem to solve

Day 11/12: EdPuzzle on Solving

- The EdPuzzle will give the students an opportunity for the students to get some practice solving while in the middle of the video
- The video will begin with directions on how to solve the simple problems
- The students will be given a simple problem to solve
- They will then be given directions for harder problems and then be asked to work a harder problem

Day 13/14: Test Review Days

- The students will each be assigned to a station to start
- Each station will be relevant to a section from the unit
 - Station 1: Horizontal and Vertical Asymptotes
 - Station 2: Adding and Subtracting
 - Station 3: Multiplying and Diving
 - Station 4: Graphing
 - Station 5: Solving
- It will not matter which station the students start with, but it will give them an opportunity to work with each
- Possible activities at each station
 - Socratic
 - Desmos
 - Solution stations

Day 15/16: Test Days

- *Reap = 60 points*
- *SMWYK/SMHYG 6.1-6.6 = 60 points*

Technology Standards

Technology Problem-Solving and Decision-Making Tools

- Students use appropriate technology resources for solving problems and making informed decisions
- Students employ technology for real world problem solving
- Students evaluate the technology selected, the process, and the final results through the use of informed decision-making skills

Technology Productivity Tools

- Students use technology tools to enhance learning, increase productivity, and promote creativity

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- Students use productivity tools to work collaboratively in developing technology-rich, authentic, student-centered products

Technology Research Tools

- Students use appropriate technology to locate, evaluate, and collect information from a variety of sources
- Students use technology tools to process data and report results
- Students evaluate and select new information resources and technological innovations based on the appropriateness to specific tasks

Basic Operations and Concepts

- Students demonstrate a sound understanding of the nature and operation of technology systems
- Students are proficient in the use of technology

Communication with Parents

To Be Sent At The Beginning of My Unit

Dear Parent of _____,

My name is Ms. Sanford and I am Mrs. Gilchrist's student teacher. I am writing this email to inform you that I will be teaching the next unit of instruction to your student. The unit I will be teaching is on rational functions, and I have some really exciting plans for activities and learning outcomes; I believe your child will really enjoy this unit! I want to give you a little bit of information on my unit, and provide any explanations of questions you may have regarding your child's learning during these next few weeks.

As I said previously, this unit is on rational functions and there are six sections associated with this unit. The sections of the unit are as follows:

- 6.1: Vertical Asymptotes and Domain
- 6.2: Horizontal Asymptotes
- 6.3: Graphing
- 6.4: Adding and Subtracting
- 6.5: Multiplying and Dividing
- 6.6: Solving

I plan to use the same assessment techniques that you are used to with Mrs. Gilchrist, meaning I will have a reop on Schoology for each section, and I will have a unit assessment at the end of the unit. However, during my unit I plan to change up the activities slightly. The students will still be familiar with my activities, as I do not plan to stray too far from their comfort zone, but I do hope to challenge them in new and exciting ways! If you have any questions about my unit or teaching style, please feel free to reach out to me at msanfo6@lsu.edu. I look forward to working with your students and helping them continue to succeed for the rest of the semester!

Sincerely,

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Mary Margaret Sanford

To Be Sent During Covid-19

Good morning Parents,

I hope you are all adjusting to this new way of life. Your child's learning and success is my primary concern and, because of the quarantine, I have decided to postpone Unit 6. This unit is historically more challenging for students to learn, so I believe it is best fit that we work on Unit 7 during this time of virtual learning. Unit 7 will be focused on Trigonometry.

The first week will be an orientation period for the students. The students will be asked to fill out a Google Form asking them how they are adjusting to life at home full-time. They will also be given a practice ACT test with solution videos in order to gain extra practice they were unable to get from taking the exam. The second week of this quarantine period will be where the students begin learning Unit 7. This unit will consist of four sections. The first will be focused on Unit Circle coordinates (including sine and cosine) and the second will be Radian measurements of the unit circle. These two sections will be taught in Week 1 of learning, while Week 2 will be focused on the second two sections: tangent and co-functions. The students will hopefully be able to come back to school after we finish Unit 7, but if not, we will move into Unit 6 from there. Each Monday the assignments for the week will be posted on Google Classroom. We anticipate that the students will spend an hour a week on Algebra II assignments, so they are easily completed in one day.

As you may know, we will not assess the students during this virtual learning time. All of the assignments given during this period will be optional, but they are strongly encouraged. Hopefully, we will return to school and the students will be able to take assessments on Unit 6 and 7, as well as the makeup reaps from previous units, as they did last semester. There will be office hours each Monday and Wednesday from 11-12 if the students have any questions, and as always, if you have any questions feel free to reach out to me at msanfo6@lsu.edu.

Have a great quarantine,

Mary Margaret Sanford

Daily Objectives

6.1: Vertical Asymptotes and Domain

- SWBAT: state the domain of a rational function
- SWBAT: state the vertical asymptotes of a rational function

6.2: Horizontal Asymptotes:

- SWBAT: state the horizontal asymptote of a rational function

6.3: Graphing

- SWBAT: state the inputs and outputs of a rational function

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- SWBAT: identify horizontal and vertical asymptotes

6.4: Multiplying and Dividing

- SWBAT: multiply a rational function
- SWBAT: divide a rational function

6.5: Adding and Subtracting

- SWBAT: add rational polynomial functions
- SWBAT: subtract rational polynomial functions

6.6: Solving

- SWBAT: Solve a rational function

Proficiency Requirements for Each Standard

	<u>6.1</u>	<u>6.2</u>	<u>6.3</u>	<u>6.4</u>	<u>6.5</u>	<u>6.6</u>
<u>Basic</u>	Given vertical asymptotes, state the domain	State the Horizontal Asymptote from a graph	State the x and y-intercepts of a function	Solve a multiplication or division problem in monomial form	State the least common divisor	Solve a rational equation by cross multiplying
<u>Proficient</u>	State the vertical asymptotes and domain from the graph	State BETC (Bottom Equals Top, leading Coefficients) and BOB0 (Bigger on Bottom, y=0)	Determine Vertical and Horizontal Asymptotes	Solve a multiplication or division problem in polynomial form	Find the LCM of a binomial	Solve a rational equation with a term +/- a number on one side, and a single term on the other
<u>Mastery</u>	State the V.A and domain from the function	State the Horizontal Asymptote of a function when the denominator is of higher power	Determine the Domain and Range of a graph	Defend/Critique an argument related to multiplying/dividing rational expressions	Add/Subtract rational monomials	Solve a rational equation with two terms on one side and a single term on the other
<u>Advanced</u>	Correct a students' work	State the Horizontal Asymptote of a function when the powers are the same	Graph a rational expression	Find the error in a student's work	Add/Subtract rational polynomials	Find the error in a problem

Lesson Plans

Week of March 16

	Monday/Wednesday (90 min)	Tuesday	Thursday/Friday (90 min)
Standard	LSSM.A2:F-IF.B.4	ACT Day	LSSM.A2:F-IF.B.4
Objective	The learners will engage in SOW 6.1 and be able to answer the ground sample correctly to achieve mastery.		The learners will identify vertical asymptotes and the domain from the function and graph to achieve mastery.
Assessment/ Evaluation	Pre-Test Desmos		Socrative 6.1 Reap
Teacher Resource/ Materials	PowerPoint Schoology Desmos Doceri Google Classroom		PowerPoint Schoology Desmos Doceri Google Classroom
Student Resource/ Materials	Schoology Show Me What You Know Sow Video Google Classroom		Schoology Desmos Sow Video Google Classroom
Technology	Chromebook Calculator		Chromebook Calculator
Accommodations/ Modification	Extended time Some students are given notes before they watch the video with blanks to fill in		Extended time Some students are given notes before they watch the video with blanks to fill in
Learning Styles	Auditory/Visual/Written		Auditory/Visual/Written
Procedure/ Activities	<p>(45) The students will take the Unit 6 pretest, which is the same as the SMHYG exam.</p> <p>(5) The students will begin the unit with a discovery on Desmos. The Desmos activity will begin with a comparison of a logarithmic function with an exponential function to look at asymptotes. They will then use a slider to discover which part of the rational function creates an asymptote. Why does the graph never touch the asymptote?</p>		<p>(15) At the beginning of class, if students have not finished the Desmos activity, they will go back in and complete more problems identifying asymptotes and domains.</p> <p>(15) When the students have completed the Desmos assignment, they will engage in Socrative for extra practice. The Socrative assignment will have questions on vertical asymptotes and domains for the students to</p>

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(30) After finishing the slider activity, the students will engage in the [SOW video](#). The Sow Video contains PKN, connections from the desmos, and new information the students will need to complete the rest of the desmos.

- I anticipate the students will struggle with understanding how to split up the domain of the rational functions with asymptotes. **Why can't we just say the domain is from $(-\infty, \infty)$?**
- The students will probably struggle with understanding how to factor out the denominator to get to the asymptote. **Can you state, in your own words, the process of factoring the denominator in order to obtain an asymptote?**

At the end of the Sow Video, the students will be asked to complete the sentence "I learned..." as a way to summarize the lesson from the day. The teacher will walk around the classroom with a clipboard [checking off](#) each student that completed the "I learned..." sentence.

(5) After the video, the students will go back into the [Desmos](#) activity and complete more problems identifying asymptotes and domains.

Higher Order Questions:

- Why does the graph never touch the asymptote?
- Why can't we just say the domain is from $(-\infty, \infty)$?
- Can you state, in your own words, the process of factoring the denominator in order to obtain an asymptote?

practice all types of questions that could be on their reap check.

(5) In the middle of the Socratic assignment, I will perform an Irrigation, where I will pick a couple of questions that less than 50% of the students are getting correct and will go over them in front of the entire class. This will allow the students who understand the material to teach their peers, and give the students who do not understand the question to learn the information without asking me for help specifically.

- I anticipate that the students will struggle with the domain questions the most. **How would this equation look as a graph? Can you draw the vertical asymptote as a graph? What would this make the domain be?**

(15) After the irrigation, the students will resume working on the Socratic assignment. If I notice that there are more questions that have less than 50% correct, I will perform another quick irrigation. However, if the students seem to be understanding the problems, we will move on to the next assignment

(10) After completing the Socratic assignment, the students will take a [reap check on 6.1](#).

(10) After completing the reap check, the students will complete an [exit ticket](#) on 6.1, where they will be asked how they feel about the lesson, where they feel they need more help, etc.

(10) After the reap check, the students will watch the [Sow video for 6.2](#) which is on Horizontal Asymptotes.

- I think the students will understand the differences between solving for a horizontal asymptote and solving for a vertical asymptote mixed up.
- I anticipate the students will also struggle with when to make the horizontal asymptote equal zero or a number

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		<ul style="list-style-type: none">- Compare and contrast the process of finding a horizontal and vertical asymptote.- Why are there different methods of solving for both asymptotes? <p>(5) At the end of the Sow Video, the students will be given a Ground Sample to solve. The teacher will walk around the classroom checking off that the students completed the ground sample. I will take five minutes after the ground sample to explain the answer to the question to the whole class</p> <ul style="list-style-type: none">- This will provide an opportunity for students to ask questions in front of the whole class- This also gives me an opportunity to catch everyone up who is behind. If a few students are struggling with the ground sample, I can solve it in front of the class so everyone can move on to the next activity. <p><u>Higher Order Questions:</u></p> <ul style="list-style-type: none">- Compare and contrast the process of finding a horizontal and vertical asymptote.- Why are there different methods of solving for both asymptotes?- How would this equation look as a graph? Can you draw the vertical asymptote as a graph? What would this make the domain be?
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Unit 6: Rational Functions

Week of March 23

	Monday (50 min)	Tuesday/Wednesday (90 min)	Thursday/Friday (90 min)
Standard	LSSM.A2:F-IF.B.4	LSSM.A2:F-IF.C.7	CCSS.A2:APR.D.7
Objective	The students will identify horizontal asymptotes and slants from the function in the graph to achieve mastery.	The students will state the inputs, outputs and given a function and match it to a graph to achieve mastery.	The students will be adding, subtracting rational, and stating the LCM to achieve mastery.
Assessment/Evaluation	Socrative Desmos 6.2 Reap	Desmos Card Sort 6.3 Reap	6.4 Reap
Teacher Resource/Materials	PowerPoint Schooly Socrative Desmos Doceri Google Classroom	PowerPoint Schooly Desmos Card Sort Doceri Google Classroom	PowerPoint Schooly Doceri Google Classroom
Student Resource/Materials	Schooly Desmos Socrative Sow Video Google Classroom	Schooly Desmos Card Sort Sow Video Google Classroom	Schooly Desmos Solution Stations Sow Video Google Classroom
Technology	Chromebook Calculator	Chromebook Calculator	Chromebook Calculator
Accommodations/Modification	Extended time Some students are given notes before they watch the video with blanks to fill in	Extended time Some students are given notes before they watch the video with blanks to fill in	Extended time Some students are given notes before they watch the video with blanks to fill in
Learning Styles	Auditory/Kinesthetic/Visual	Auditory/Kinesthetic/Visual	Auditory/Kinesthetic/Visual
Procedure/Activities	(10) The class will begin with a recall activity called Quiz-Quiz-Trade . The activity will be a recall of the information the students have learned from 6.1, namely vertical asymptotes and domain of rational expressions. The process of how the activity will work is below: <ul style="list-style-type: none"> - Each student will receive a card 	(10) The students will begin the day with a SOW video on how to find the range of a rational expression. This will help them when they are attempting to graph, and they have never seen anything similar to this, so it will be direct instruction. <ul style="list-style-type: none"> - I expect the students will struggle with finding the inverse of an expression 	(10) The students are going to begin with a recall activity to jog the students' memories of skills that will be necessary for the lesson that day. (50) The students will spend a large portion of the class period working through our video activity. For this activity, the main video will be split into a video on multiplying and

Unit 6: Rational Functions

<ul style="list-style-type: none"> - The students will walk around the room quizzing each other on their cards - There are twelve different cards, so once each student has answered eleven questions, the activity is over and the students can return to their seats <p>(10) The students have already engaged in a Sow video for 6.2 last class, so they will move straight into the Desmos activity. The desmos activity will serve as a refresher of what the students learned from the 6.2 video, as well as an introduction into the socrative activity.</p> <p>(15) Upon completing the desmos activity, the students will engage in a Socrative to practice problems on finding horizontal asymptotes.</p> <ul style="list-style-type: none"> - I anticipate students will forget what the horizontal asymptote is if the powers are different between the numerator and denominator - What are the differences between a horizontal asymptote and a vertical asymptote? - Why are there different methods of solving for both asymptotes? <p>(10) After the students have completed the Socrative assignment, they will take their reap assessment for 6.2.</p> <p>(5) At the end of the lesson, the students will complete an exit ticket on 6.2, where they will be asked how they feel about the lesson,</p>	<ul style="list-style-type: none"> - The students will be required to show me their notes after they have finished the video, so I know they sufficiently tried to understand the material <p>At the end of the video, the students will be asked to complete the sentence “I learned...” as a way to summarize the notes from 6.3. The teacher will walk around the classroom with a clipboard checking off each student that completed the “I learned...” sentence.</p> <p>(20) After the first video, the students will work on a desmos activity that will serve as both a recall of asymptotes, as well as an introduction to the lesson on graphing.</p> <ul style="list-style-type: none"> - At Slide 8, titled “Predict #1” the teacher will pause the students to discuss what they think will happen to the graph if we changed the domain from 5-10 - If the students struggled with slide 8, the teacher should also pause at slide 10, titled “Predict #2” so the students can have more discussion <p>(15) After completing the Marbleslide activity, the students will engage in a SOW 6.3 video to clarify points they did not understand from the desmos. How can we connect these expressions to the end behavior of a polynomial?</p> <p>(5) At the end of the Sow Video, the students will be given a Ground Sample to solve. The teacher will</p>	<p>dividing monomials, and a video on multiplying and dividing polynomials. After the students complete each of the videos, there will be a set of problems that they will need to solve in order to move on to the next video. The problems are organized in accordance with the videos. There are a set of problems that the students will complete after the first video and a set they will complete after the second video</p> <ul style="list-style-type: none"> - I anticipate the students will struggle with multiplying and dividing polynomials. I do not think they will see that they can factor the expressions to possibly cancel out terms before multiplying. Why do we have to flip and multiply when dividing rational expressions? - I also think the students will struggle with factoring, as that seems to be a common issue throughout units. <p>(5) The students will be required to show me their work on the solution stations from the first video before they can move onto the next video. This way, I will be able to check off that the students are understanding the easier problems, before they move into the harder ones. There is also a ground sample at the end of video 2, so I will check off students for that as well.</p> <p>(10) When most students have begun working on the first set of problems, I perform an irrigation and go over a problem at the board that many students are struggling with. I will do the same with the</p>
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Unit 6: Rational Functions

where they feel they need more help, etc.

Higher Order Questions:

- What are the differences between a horizontal asymptote and a vertical asymptote?
- Why are there different methods of solving for both asymptotes?

walk around the classroom [checking off](#) that the students completed the ground sample. If 50% of students are struggling with the ground sample, then I will go over the solution in front of the class to clarify.

(25) The students will then complete an activity where they will [match cards](#) to their functions and graphs in groups of 5-7

- I anticipate that students will struggle with the meaning of “horizontal” and “vertical” and mix up how to draw each line
- I also expect a lot of calculator error with the inputs and outputs of the expression to graph it

To address these struggles, we will have a class discussion on how to find horizontal and vertical asymptotes as well as how to input values into the calculator.

- I will prepare the groups before class to make sure students will work well together.
- If high-performing students are placed with low-performing students, I want to make sure the high students will help the low students and not just do all of the work

(10) After this activity, the students will take the [6.3 Reap Check](#).

(5) At the end of the lesson, the students will complete an [exit ticket](#) on the section, where they will be asked how they feel about the lesson, where they feel they need more help, etc.

second set of problems.

(10) The students will then take the [reap check for 6.4](#) to end the class period.

(5) At the end of the lesson, the students will complete an [exit ticket](#) on the section, where they will be asked how they feel about the lesson, where they feel they need more help, etc.

Higher Order Questions:

- Why do we have to flip and multiply when dividing rational expressions?

Unit 6: Rational Functions

		<p><u>Higher Order Questions:</u> - How can we connect these expressions to the end behavior of a polynomial?</p>	
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Week of March 30

	Monday (50 min)	Tuesday/Wednesday (90 min)	Thursday/Friday (90 min)
Standard	CSS.A2:APR.D.7 LSSM.A2:A-REI.4b	CSS.A2:APR.D.7 LSSM.A2:A-REI.4b	CCSS.A2:APR.D.7 LSSM.A2:A-REI.4b
Objective	The students will engage in the Sow video on adding and subtracting rational polynomials, and correctly answer the ground sample, to complete mastery.	The students will add and subtract rational polynomials in order to achieve mastery.	The students will solve the rational equation and identify the error to complete mastery.
Assessment/ Evaluation	Ground Sample	Solution Stations Lockbox Card Sort Reap 6.5	Solution Stations Reap 6.6
Teacher Resource/ Materials	PowerPoint Schoology Desmos Screencast-o-matic/Doceri Google Classroom	PowerPoint Schoology Desmos Screencast-o-matic/Doceri Google Classroom	PowerPoint Schoology Desmos Screencast-o-matic/Doceri Google Classroom
Student Resource/ Materials	Sow video ground sample Google classroom Schoology	Google classroom Schoology Lock box Solution stations	Sow video Solution stations Google classroom
Technology	Chrome book Calculator	Chrome book Calculator	Chrome book Calculator
Accommodations/ Modification	Extended time Some students are given notes before they watch the video with blanks to fill in	Extended time Some students are given notes before they watch the video with blanks to fill in	Extended time Some students are given notes before they watch the video with blanks to fill in

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Learning Styles	Auditory/Kinesthetic/Visual	Auditory/Kinesthetic/Visual	Auditory/Kinesthetic/Visual
<p>Procedure/Activities</p>	<p>(25) The students will begin class with a recall activity. The students will be given individual marker boards and will be asked to hold up their answer to each question. After each slide, I will allow a maximum of two minutes for students who got the correct answer to explain the solution to students who did not get the correct answers.</p> <p>(20) The students will engage in the SOW video for 6.5, adding and subtracting rational expressions.</p> <ul style="list-style-type: none"> - This section is supposed to be one of the hardest of the unit, so I anticipate a lot of problems both on this day and the following two days, as we finish up the lesson - I see struggles with finding the LCM, as well as combining like terms - How do you find LCM? - How can we connect these types of problems to the real world? <p>(5) At the end of the Sow Video, the students will be given a Ground Sample to solve. The teacher will walk around the classroom checking off that the students completed the ground sample. If 50% of students are struggling with the ground sample, then I will go over the solution in front of the class to clarify. I anticipate the students will struggle with this ground sample.</p>	<p>(5) The students will walk into class and we will have a quick group discussion on what the video talked about from Monday. We will discuss the following topics</p> <ul style="list-style-type: none"> - How to find the LCM - Distributing a negative - Multiplying the numerators by the LCM <p>(65) The students will complete a rotation station activity. The students will have 20 minutes at each station and the stations will be a lock box, a card sort, and a solution station.</p> <ul style="list-style-type: none"> - I think the students will struggle with the lock box the most. I think it will mostly be typing error, but I think they will struggle with going through the problems step-by-step - The students will be grouped according to the row they sit in. Their seating assignments are already an effort to minimize talking, so each group will consist of two rows of students. - The lockbox activity will help show students where they are making mistakes specifically. It is broken down into step-by-step instructions, so students will be able to see where exactly they are messing up when attempting to add/subtract. This is an individual activity, so the students can see their struggles without worrying about where their peers are struggling. How do you find LCM? 	<p>(15) The students will begin class with a recall of the unit, going over every topic covered since the unit began. This activity will be completed using marker boards and the students will hold up their answers for only me to see. This will create anonymity amongst the students and will allow me to see where misconceptions may still lie</p> <p>(15) They will then engage in the SOW video for 6.6, which discusses solving rational equations. The SOW video will go through one example of each type of question so the students can see an example worked out of everything they will see.</p> <p>(5) At the end of the Sow Video, the students will be given a Ground Sample to solve. The teacher will walk around the classroom checking off that the students completed the ground sample. If 50% of students are struggling with the ground sample, then I will go over the solution in front of the class to clarify.</p> <p>(30) After engaging in the SOW video, the students will complete solution stations. The solution stations will have questions that cover each type of question asked on the reap check.</p> <ul style="list-style-type: none"> - I anticipate that the students will struggle with the questions that involve a term +/- a number = a term. I think they will try to cancel out the denominator before they make the whole number

Unit 6: Rational Functions

	<p><u>Higher Order Questions:</u> - How do you find LCM? - How can we connect these types of problems to the real world?</p>	<ul style="list-style-type: none"> - The card sort will allow students to work together to solve problems and put them in the correct spots. The students will be able to peer coach each other and compare strategies for solving. The solutions will be in a manilla folder, and the students will have to show me their final results before they can check the solutions. How can we connect these types of problems to the real world? - The solution station is again an individual activity, but it takes in components of the previous two activities. The students will be able to see where their mistakes lie, but they can also discuss solutions with their peers and get help from each other. <p>(10) After the rotation stations, the students will engage in the 6.5 reap check.</p> <p>(5) At the end of the lesson, the students will complete an exit ticket on the section, where they will be asked how they feel about the lesson, where they feel they need more help, etc.</p> <p><u>Higher Order Questions:</u> - How do you find LCM? - How can we connect these types of problems to the real world?</p>	<p>have a common denominator</p> <ul style="list-style-type: none"> - How can we get rid of a fraction if it is just a whole number, not an expression? <p>The students will be able to work with their classmates to peer coach each other and explain reasoning for certain solutions. The solutions will be posted on either side of the room with the steps written out so students can see exactly where they made a mistake. This allows the students to self-grade and gain further insight into a topic. If students are still confused they are able to discuss with me or with a peer.</p> <p>(10) Throughout the Solution Stations, if many students are confused about the same problem, I will go over it in front of the entire class in an irrigation. This will allow everyone to see it all at once and ask questions that other students may not realize they have. If there is a small number of students confused about a problem, I will conduct a campfire at the board so I do not disrupt the entire class. This will allow more individualized help for the students.</p> <p>(10) After completing the solution stations, the students will take their reap check for 6.6.</p> <p>(5) At the end of the lesson, the students will complete an exit ticket on the section, where they will be asked how they feel about the lesson, where they feel they need more help, etc.</p>
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Unit 6: Rational Functions

Higher Order Questions:
 - How can we get rid of a fraction if it is just a whole number, not an expression?

Week of April 6

	Monday/Tuesday	Wednesday/Thursday	Friday
Standard			NO SCHOOL
Objective	The students will be self assessing on rational functions under standard testing conditions. The students will check their self assessment against a key to determine areas of improvement and practice those in order to grow.	The students will show their growth under standard testing conditions.	
Assessment/Evaluation	SMWYK Grow Stations	SMHYG	
Teacher Resource/Materials	Reaps for individual standard results.	Reaps for individual standard results.	
Student Resource/Materials	Video notes from 6.1-6.6 standards/ “show me what you know” practice test.	Video notes from 6.1-6.6 standards/ “show me how you grow” practice test.	
Technology	Calculator Chromebook	Calculator Chromebook	
Accommodations/Modification	Extended time for students that have ADHD or ADD or any accommodation that requires them extended time.	Extended time for students that have ADHD or ADD or any accommodation that requires them extended time.	
Learning Styles	Visual, Kinesthetic, Auditory	Visual, Kinesthetic, Auditory	
Procedure/Activities	The students will complete the SMWYK and a completed SMWYK will be posted on Google Classroom. This will give students the opportunity to work with their peers on a practice exam. The format of the SMWYK is the exact same as the format of	The students will take their unit 6 exam, or SMHYG , that was also given to them as a pre-test at the beginning of the unit. The exam will have one question for each proficiency requirement, totalling to be 24 questions. The exam will be worth 60 points in total, at 10 points per section. The	

Unit 6: Rational Functions

the SMHYG, giving the students a chance to know what the exam will look like.

The students will also work through [grow stations](#), which will have one new question related to each of the proficiency requirements for the exam. This is optional for the students; they should only work on problems with which they are struggling on the SMWYK.

There will also be a review reap. For this quiz, one random question from each bank of the unit will be pulled and the students can take it as a practice assessment. There are 22 banks from the entire unit in total, so each reap check will be 22 questions. This will be open for the students until the day of the exam, so they can take it as many times as they need to for extra practice. Since there are 22 questions, and at least 4 options for each question, it is very unlikely that the students will get the same quiz twice. This will give the opportunity for the students to practice their skill set and accuracy in problem solving.

At the end of the class period, the students will complete an [exit ticket](#) on the unit, where they will be asked how they feel about the unit as a whole, where they feel they need more help, etc. This will be a final evaluation to determine where students are struggling the most. This will also give students an idea of where they need to spend time studying before the test.

students will be assessed on many levels of Bloom's Taxonomy in an attempt to determine their levels of understanding.

After students have completed the exam, they will be given an [exit ticket](#) that asks them how confident they felt before and after the exam. It will also ask them how well I taught the material, where I could have improved instruction, etc. This exit ticket is solely for my benefit so I can see where I need to improve as a teacher.

Assessments and Descriptions

The assessments below are listed in order of use in the Unit. Each assessment will begin with a link to the assessment (or a link to images of the assessment) with a description of the assessment following the link.

Unit 6 SMHYG (Show Me How You Grow)

This assessment will serve as both the pre-assessment for the unit, as well as the Unit 6 exam, or post assessment. Each of the sections covered in the unit will be sectioned off on the exam so the students know what sections they are working on. The levels of each question are also identified, in an attempt for the students to understand where they are at in their learning. This identification will also help me understand how well the students have understood each standard. For example, if the students are all missing the basic questions of a section, then I will know that they did not understand the lesson as well as I thought they did, and that I should have been more thorough during my lesson.

6.1 Reap

This is an end-of-section assessment that the students will take at the end of the class period. The assessment will be four questions that cover the types of questions the students learned in class. Since this section is on vertical asymptotes and domain, the students will have one question from the following banks: State the asymptote (binomial), State the asymptote (polynomial), State the domain (equation), and State the domain (graph). This assessment is on Schoology, so the website randomly picks a question from each bank for the students to work.

6.2 Reap

This is an end-of-section assessment that the students will take at the end of the class period. The assessment will be four questions that cover the types of questions the students learned in class. Since this section is on horizontal asymptotes and slant for honors, the students will have one question from the following banks: $HA = 0$, $HA = \#$, Slant Asymptote (Honors), HA from Graph, and Range. This assessment is on Schoology, so the website randomly picks a question from each bank for the students to work.

6.3 Reap

This is an end-of-section assessment that the students will take at the end of the class period. The assessment will be four questions that cover the types of questions the students learned in class. Since this section is on graphing, the students will have one question from the following banks: Input/Output – 1 asymptote, Choose the graph, and Input – 2 asymptotes. This assessment is on Schoology, so the website randomly picks a question from each bank for the students to work.

6.4 Reap

This is an end-of-section assessment that the students will take at the end of the class period. The assessment will be four questions that cover the types of questions the students learned in class. Since this section is on multiplying and dividing, the students will have two questions from the

Unit 6: Rational Functions

multiplying bank and the dividing bank. This assessment is on Schoology, so the website randomly picks a question from each bank for the students to work.

6.5 Reap

This is an end-of-section assessment that the students will take at the end of the class period. The assessment will be four questions that cover the types of questions the students learned in class. Since this section is on graphing, the students will have one question from the following banks: Simplify with common denominator, Binomial – finding LCM, Simplifying – Adding, and Simplifying – subtracting. This assessment is on Schoology, so the website randomly picks a question from each bank for the students to work.

6.6 Reap

This is an end-of-section assessment that the students will take at the end of the class period. The assessment will be four questions that cover the types of questions the students learned in class. Since this section is on solving, the students will have one question from the following banks: Cross Multiplying, Term +/- #, Term +/- Term, and Two Terms = Two Terms. This assessment is on Schoology, so the website randomly picks a question from each bank for the students to work.

Unit 6 SMWYK (Show Me What You Know)

This assessment will be a practice test given to the students a couple of days before the unit assessment. This assessment is in the exact format of the test, with different questions. The students receive this so they know what the format of the test will look like, and what types of questions they should expect from the assessment.

Unit 6 SMHYG (Show Me How You Grow)

This assessment will be the exact same assessment given to the students as a pre-test. The students will take this pretest as their unit 6 assessment, as well as a data point for me to analyze their growth in the unit after I have taught.

Sources

<http://www.math.utah.edu/~wortman/1050-text-rf.pdf>

<https://courses.lumenlearning.com/waymakerintermediatealgebra/chapter/15-3-1-rational-formulas-and-variation/>

<https://www.sparknotes.com/math/calcab/applicationsofthederivative/section7/>

<https://www.onlinemathlearning.com/asymptote.html>